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Stringy Higgs mechanism and the fate of open strings

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Abstract: We propose a refinement of the physical picture describing different vacua in bosonic string theory. The vacua with closed strings and open strings are connected by the string field theory version of the Higgs mechanism, generalizing the Higgs mechanism of an abelian gauge field interacting with a complex scalar. In accordance with Sen's conjecture, the condensation of the tachyon is an essential part of the story. We consider this phenomenon from the point of view of both a world-sheet sigma-model and the target-space theory. In the appendix the relevant remarks regarding the choice of the coordinates in the background independent open string field theory are given.

Keywords: Bosonic Strings, D-branes, Superstring Vacua.

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Contents

1. Introduction

Since the conjectures regarding the vacuum structure of open string theory were made by Sen [1, 2], considerable interest has been devoted to its verification as well as study of the consequences. The studies of [3, 4, 5] demonstrated that the approach based on background independent open string field theory of [6, 7] can be very effective in addressing the question of tachyon condensation since one can find the exact expression for the string field theory action order by order in a derivative expansion of space-time fields. In this standard way of treating the low energy lagrangian, the exact answer already in the two derivative approximation provides important information and allows one to demonstrate the validity of Sen's conjectures.[1] Further generalizations were considered in [9, 10] for the case of a constant B-field and in [11] for the case of superstrings.

According to the Sen's conjectures for the open bosonic string theory, the process of tachyon condensation leads to a new vacuum state which doesn't contain any open string states. It is naturally identified with the perturbative vacuum of the closed bosonic string theory. This gives the convincing support to the idea that *the perturbation theories of open and closed strings are expansions in some background independent universal theory around the different vacua.*

[1]It is very interesting to note that in [8], motivated by rather different ideas, the same tachyon lagrangian, both in the bosonic and superstring case, was proposed as a toy model that mimics the expected properties of tachyon condensation.

We would like to propose a further explanation of the connection between open and closed strings, guided by the symmetries of string theory. It turns out that in the process of learning how the electro-magnetic field disappears as a result of tachyon condensation in the approach of background independent open string field theory, we come to an important understanding regarding the properties of string theory in general: the connection between the two vacua is a close relative of the standard Higgs description of two types of vacua in the theory of an abelian gauge field coupled to a complex scalar field. The perturbative vacuum of the closed strings plays the role of the invariant vacuum, and the perturbative vacuum of the open strings is the vacuum with the spontaneously broken symmetry.

In the vacuum with the spontaneously broken symmetry the open string tachyon is similar to the absolute value of the complex scalar field in the field theory example, and the role of the phase is played by the other fields in the open string spectrum. In this analogy the gauge transformations of the open string fields are similar to the identification of the angular variable $\theta \sim \theta + 2\pi$. This is the manifestation of the particular choice of the coordinates and not of the topology of the configuration space. As the consequence, for the other coordinates (e.g. natural variables in the closed string theory) there is no trace of these open string gauge symmetries. This serves as the explanation of the absence of the open string gauge fields after tachyon condensation. We note that the tachyon potential in open string field theory is very different from the standard field theory example $V(\phi) = (|\phi|^2 - \eta^2)^2$. In particular the vacuum with the spontaneously broken symmetry is unstable. Thus the perturbative expansion around the symmetric vacua is in terms of closed string degrees of freedom. The other degrees of freedom are suppressed in the symmetric vacuum by the infinite effective mass. From the point of view of the closed string theory world-sheets, these hidden degrees of freedom correspond to non-smooth world-sheet deformations.

The main suggestion we make in this paper is closely related to the search for closed strings inside the field theory of open strings; this has been a challenge for many years (see e.g. [12]) and more evidence has occurred recently [13] in the view of developments related to D-branes and matrix theory [14].[2] Note also that a related qualitative picture of the disappearance of the open string gauge fields and the emergence of the closed strings was presented in [15] and [5].

In section 2 we consider the truncated version of the open/closed string theory (the approximation we use is rather close to the considerations in [16]). We demonstrate that the process of the tachyon condensation closely follows the Higgs mechanism in the theory of a U(1)-gauge field interacting with a complex scalar (of

[2]It has been proposed in [13] (based on several evidences extracted from matrix stings on D1 brane in IR limit) that in background independent open string field theory (D9 or D25) there should exist a solitonic solution corresponding to fundamental, Nambu-Goto, closed string and thus open string field theory can serve as the definition of full, self-consistent theory of interacting open and closed strings.

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course with an appropriate map between fields of abelian Higgs model and truncated string field theory). Based on this analogy we give a qualitative picture of the new vacuum. The considerations of this section suggests that one should think about the Higgs phenomena taking place in the space of loops instead of space-time (when all stringy modes are included). If the gauge bundles over the space-time play a prominent role in the usual quantum theories of fields, the proper analogs in the theory of strings are gerbes with 2-connections. Thus we suggest that the basic mechanism behind the open/closed string transformation is, in the nutshell, the Higgs phenomenon for gerbes.

The fact that the B field absorbs the gauge field in the Higgs mechanism, familiar from supergravity theories, was know from early days of string theory [17], and in the context of Born-Infeld action [18] (for some early studies of tachyon condensation see [19]). The appearance of the gauge group $\mathrm{SU}(N)$ instead of $\mathrm{U}(N)$ in the AdS/CFT correspondence [20] possibly has the same origin.

It is well known that the separation of closed string and open string degrees of freedom is rather ambiguous. The inclusion of the singular interactions in the open string theory gives the closed string states in the loop expansion. On the other hand, one could use the smooth open string vertices at the cost of explicit introduction of closed string fields. We claim: after tachyon is properly included in the picture and when it condenses, the coordinates with explicit closed strings become more appropriate and closed strings appear as dynamical variables. We conjecture that all open string fields (except tachyon) are "angle" variables and correspond to gauge parameters for corresponding closed string fields.

In section 3 we begin to analyze this interpretation directly in the sigma model approach. The transformation of the open strings (2d surfaces with boundaries) to the closed strings (2d surfaces without boundaries) is a "geometric" one and this leads us to believe that the sigma model approach could provide the basic insight into the question of the disappearance of open strings. In the new vacuum, the condensate of the tachyon zero mode becomes infinite, and due to the general prefactor e^{-RT_0} for each open string loop, forbids a non-zero boundary on the world-sheet.[3] In this section we discuss the connection between these two vacua in terms of the 2d quantum theory on the world-sheet and find the picture of spontaneous symmetry breaking similar to the one discussed in section 2.

In order to gain more information in section 4 we use the open/closed string field theory due to Zwiebach [21] to verify the off-shell connection between open string fields and closed string gauge parameters.

In the appendix we discuss two important questions regarding the role of the choice of the proper coordinates in the background independent open string field theory action.

[3]Related question was asked by S. Shenker and we thank him for sharing it with us.

To conclude this introduction we would like to mention that the question: *what is exactly the space of boundary field theories in 2d?* seems to be of the fundamental importance in the open/closed string relation.[4] In this notes we give just first steps towards the possible answer.

2. Stringy Higgs mechanism: target-space approach

In order to study the role of higher spin fields in the process of tachyon condensation we start with the truncated field content in the open/closed string theory. Thus we consider the metric G, two-form gauge field B, open string U(1) gauge field A and open string tachyon T.

Let us remark that in general the inclusion of the full spectrum of open and closed strings in the action leads to the over-counting of the degrees of freedom and their contributions to amplitudes. But here we are looking at the truncated open string theory and this consideration is legitimate.

In background independent open string field theory we start from the world-sheet theory defined on the disk with the operator $e^{-\int d\theta(T+AdX+\dots)}$ inserted on the boundary (we assume that ghosts decouple). This operator is invariant under open string gauge transformations $A \to A + d\Lambda$ and global shifts $A \to A + \text{const}$. The space-time action for these fields has the form (see the appendix for the choice of the parameterization):

$$S(G,B,A,T) = S_{\text{closed}}(G,B) + \int d^{26}X\sqrt{G}\Big(e^{-T}(1+T) + e^{-T}||dT||^2 + \\ + \frac{1}{4}e^{-T}||B-dA||^2 + \dots\Big). \qquad (2.1)$$

One shall note the obvious gauge invariance of the action:

$$B \to B + da \qquad (2.2)$$

$$A \to A + a\,, \qquad (2.3)$$

where a is 1-form gauge parameter. Global symmetry corresponds to constant a.

We can consider two situations: closed string modes are fixed (non-dynamical) backgrounds and closed string modes are dynamical. As it follows from the tachyon potential, in both cases we have two types of the vacua in the theory. At the open string perturbative vacuum, $T = 0, A = 0$, we have the tachyon of finite mass and therefore this vacuum is unstable. In the theory of only open strings we also have the massless gauge field A. When we make the closed string modes dynamical (which is not necessary at the moment) — in this vacuum the gauge invariance (2.2), (2.3) allows to put A to be zero and we are left with the tachyon and mass term for the field B.

[4] This question have been raised by E. Witten over the years since [6, 7].

It appears that the open string tachyon potential has another vacuum for the infinite value of the tachyon field $T = \infty$ (in addition there are many soliton solutions corresponding to the lower D-branes but we will not discuss them here). In the new variables:

$$\Sigma = e^{-1/2T} \tag{2.4}$$

it is at $\Sigma = 0$ (unstable vacuum is at $\Sigma = 1$) and:

$$S(G, B, A, \Sigma) = S_{\text{closed}}(G, B) + \int d^{26}X\sqrt{G}\Big(\Sigma^2(1 - 2\log\Sigma) + 4||d\Sigma||^2 + \\ + \frac{1}{4}\Sigma^2||B - dA||^2 + \ldots\Big). \tag{2.5}$$

Around the critical point $\Sigma = 0$ from corresponding potential in (2.5) we conclude that the square of mass for Σ-field ("tachyon") is positive and infinite. According to Sen this should be the vacuum corresponding to the theory of the closed strings. Note that the kinetic term for the gauge field A multiplies zero, Σ^2, at this point and thus the gauge field is not well defined at this vacuum.

Last expression (2.5) immediately suggests the analogy with the quantum field theory textbook lagrangian:

$$S(\Phi, \mathcal{A}) = \int dX\left(\frac{1}{g^2}F(\mathcal{A})^2 + |d\Phi - i\mathcal{A}\Phi|^2 + \lambda(|\Phi|^2 - |\Phi_0|^2)^2\right) \tag{2.6}$$

with gauge transformations:

$$\mathcal{A} \to \mathcal{A} + d\chi \tag{2.7}$$

$$\Phi \to e^{i\chi}\Phi \tag{2.8}$$

that are analogous to (2.2) and (2.3). Here we have the abelian gauge field $\mathcal{A}$ interacting with the complex scalar field Φ and as an example the forth order polynomial potential.

In order to make the connection to string theory lagrangian (2.5) we allow ourselves to briefly review the properties of (2.6) in the variables similar to (2.5):

$$\Phi(X) = e^{i\phi(X)}H(X). \tag{2.9}$$

The action (2.6) is:

$$S(H, \phi, \mathcal{A}) = \int dX\left(\frac{1}{g^2}F(\mathcal{A})^2 + H^2|d\phi - \mathcal{A}|^2 + |dH|^2 + \lambda(H^2 - H_0^2)^2\right). \tag{2.10}$$

In the new coordinates, ϕ is identified under the shift transformation:

$$\phi \to \phi + 2\pi. \tag{2.11}$$

The appearance of the field identification (2.11) is not the manifestation of the non-trivial topology of the configuration space, but it is the artifact of the choice of the special coordinate system. For instance in terms of two scalar fields X, Y ($\Phi = X + iY$) there is no condition like (2.11).

In these variables the gauge transformations are exactly like for string theory case (2.2) and (2.3):

$$\mathcal{A} \to \mathcal{A} + d\chi \tag{2.12}$$

$$\phi \to \phi + \chi\,. \tag{2.13}$$

The stable vacuum $\Phi = \Phi_0$ is not invariant under phase-shift — it is stable and non-symmetric. When gauge field $\mathcal{A}$ is background field, we have massive scalar H and massless scalar-phase ϕ. When $\mathcal{A}$ is dynamical we can set the angular variable to zero by gauge transformations and we get massive H and massive gauge field A.

There is also unstable but symmetric vacuum — $\Phi = 0$. In radial variables for background gauge field $\mathcal{A}$ we have tachyonic field H and the phase field ϕ is ill-defined since its kinetic term multiplies zero expectation value of H-field. This is the manifestation of the fact that these variables are not well defined at this point and one should use another (non-singular) parameterization (e.g in terms of X and Y). At the same time, when gauge field becomes dynamical we can perfectly live with angular variables; the angular field ϕ is a gauge parameter for $\mathcal{A}$ - we get massive tachyonic field H plus massless gauge field $\mathcal{A}$. This is how field theory abelian Higgs mechanism looks in angular coordinates.

The analogy between (2.10) and (2.5) is quite obvious. We can "map" the variables as:

$$\Sigma \to H \tag{2.14}$$

$$A \to \phi \tag{2.15}$$

$$B \to \mathcal{A}\,. \tag{2.16}$$

The description of open string theory in terms of the fields Σ, A in (2.5) is similar to the description of abelian field theory in terms of matter fields H, ϕ. Closed string mode B is related to gauge field $\mathcal{A}$. The tachyon (more exactly the field Σ) plays the role of the "radial" component of the complex scalar field and the gauge field is the analog of the "angular" variable. The gauge field A shifts under the gauge transformation (2.3) similar to the ϕ in the previous considerations. It is interesting that the usual gauge transformation of A is the full analog of the shift symmetry (2.11).

The analogy described above taught us the following: we can study the truncated string theory system around the vacuum ($T = 0, \Sigma = 1$), $A = 0$ (unstable and non-symmetric) and the corresponding situation in abelian Higgs model is stable/non-symmetric vacuum. Otherwise, if we study the string theory around the new vacuum at $\Sigma = 0$ which is invariant with respect to closed string symmetry (2.2) ("stable"/symmetric vacuum) - it is similar to the symmetric point in the configurations space of the abelian gauge field theory example (unstable/symmetric). But from the field theory considerations we know that the description of the fluctuations around symmetric vacuum configuration in terms of the "angular" type variables and fixed background field $\mathcal{A}$ is totally inappropriate and one should use the different parameterization of the fields (for example cartesian). Now, we shall note that latter problem in abelian Higgs model is removed by choosing the correct coordinates, and by dynamical gauge field $\mathcal{A}$. If we go back to string theory example it seems that angular variables are forced on us from the beginning in truncated open/close string system and in case of fixed closed string background there is no possibility to introduce the analog of local cartesian coordinates X, Y since the analog of absolute value of $\Phi = H$ (which is Σ) and phase factor ϕ (which is A) carry different space-time spin. At the same time we can introduce non-local string field theory wave-function

$$\Psi(X(\sigma)) = e^{-\int (T(X(\sigma) + A(X)dX(\sigma) + \cdots)d\sigma} \tag{2.17}$$

which may be considered as the formal analog of the complex scalar field Φ and the 2-form B field gives the natural connection on the space of these functionals (this means that we now need to include all fields and all derivatives in space-time lagrangian which becomes the lagrangian in loop space). Thus we conclude that in the truncated string model it is forced to use non-local string field variable and assume that closed string modes are dynamical. The latter is an important conclusion since what has been claimed is that there is a new branch in open string field theory where the only dynamical degrees of freedom are closed strings. Two branches are connected by new, stable, closed string vacuum $\Sigma = 0$.

All this support the idea that at the new vacuum of the string theory there is no open string states in the spectrum and we have the theory of the closed strings instead.

It is interesting to note that the string field theory wave function (2.17) is closely related to cubic **CS** open string field theory coordinates (see [3]). Let us remember that in [3] the following relation between tachyon modes in sigma-model and cubic **CS** string field theory coordinates was proposed: consider world-sheet path integral for the disk topology and divide the disk into two equal parts with first half carrying the fixed boundary condition $X(z, \bar{z})|_{\partial D} = X_*(\sigma)$ and on second half the operator (2.17) inserted. For tachyon zero modes this leads to the relation:

$$e^{-1/2T_0} = 1 + T_0^{CS} \,. \tag{2.18}$$

We see that the cubic **CS** string field theory expansion is built around the vacuum corresponding to $T_0^{CS} = 0$. One can use the conformal transformation in order to map the disk to "1/3 of pizza" - with 120^o angle segment and glue three such wave-functions in order to get familiar cubic term in **CS** action which now becomes the disk partition function with functional (2.17) inserted on the boundary (second term in the background independent action $S = -\beta^i \partial_i Z + Z$); as far as the β-function term in the action - it is obviously obtained from the kinetic term $\Psi Q \Psi$ of **CS** cubic action (for the appropriate regularisation in sigma model).

3. World-sheet considerations

Now we would like to understand the above picture in terms of world-sheet sigma model. Consider the perturbation series expansion in the string theory with open strings. We should sum up the contributions from the arbitrary genus surfaces with arbitrary number of the holes. The contribution of each surface is given by the 2d functional integral over the fields with Neumann boundary conditions ("partition function") which has the expansion:

$$Z_{\mathrm{Total}} = \sum_{h,n} \frac{1}{n!} g^{2h-2+n} Z_{\Sigma_{g,n}} \,. \tag{3.1}$$

We will mainly be interested in the contributions of the open string loops and thus restrict ourselves by the genus zero surfaces.

$$Z_* = \sum_{n} \frac{1}{n!} g^{n-2} Z_{\Sigma_{0,n}} \,. \tag{3.2}$$

This expansion may be interpreted as the closed string partition function in the "shifted" background. Being not very precise one could say that there is an operator in 2D theory such that its insertion in the correlator simulates the appearance of the hole on the world-sheet. If we denote this operator V_H then the partition function (3.2) may be formally written as an "effective action" for the closed string in some new background:

$$Z_* = \sum_{n} \frac{1}{n!} \langle {V_H}^n \rangle = \langle e^{V_H} \rangle \,. \tag{3.3}$$

The addition of this operator to the world-sheet action deforms the vacuum of the closed string theory to the new vacuum where the world-sheets with holes are possible.

This operator may be described explicitly. Consider Hilbert space of states of the quantum fields defined on the circle of the radius R in the first quantized closed string theory. The vacuum state is define in terms of the annihilation operators in the standard way:

$$\alpha^i_n |\mathrm{vac}\rangle = 0 \,. \tag{3.4}$$

Geometrically it means that this vacuum state is induced by the functional integral over the disk. This state is invariant under various symmetries. For instance it is invariant under the gauge transformation of the 2-form field $B \to B + \mathrm{da}$ which will be important in the following consideration. The perturbative expansion around this vacuum is given in terms of the standard closed string modes. Corresponding operators are given by the polynomials over creation operators (up to the momentum factor $e^{ip_i X^i}$).

In open string diagrams one chooses Neumann boundary conditions - the normal derivative of the fields on the boundary should vanish. This is very different from the previous case and doesn't correspond to the vacuum state induced by the integral over the disk. Thus in this case we have the real hole on the string world sheet. The vanishing normal derivative is equivalent to the condition:

$$\partial_\sigma X^i_L(\sigma) = \partial_\sigma X^i_R(\sigma)\,. \tag{3.5}$$

Here $X^i_{L,R}$ are restrictions on the boundary of the chiral and antichiral components of the scalar fields. The corresponding condition on the open string vacuum state may be conveniently written in terms of the canonical momentum variables $P^i_n = 1/2\int d\sigma(\partial_\sigma X^i_L(\sigma) - \partial_\sigma X^i_R(\sigma))$ as:

$$P^i_n|\text{open}\rangle = 0\,. \tag{3.6}$$

The transformation between two states is a standard Bogolubov transformation with some operator:

$$|\text{open}\rangle = U_H|\text{vac}\rangle \tag{3.7}$$

$$U_H = e^{1/2A(\alpha,\alpha)}\,, \tag{3.8}$$

where $A(\alpha,\alpha)$ is a bi-linear form on the creation and annihilation operators. Its explicit expression is not important at the moment.

The corresponding wave function:

$$\Psi(X(\sigma)) = \langle X(\sigma)|\text{open}\rangle \tag{3.9}$$

may be considered as the classical solution of the closed string theory corresponding to the open string vacuum. To get the corresponding sigma model vertex operator we should recall that the additional moduli of conformal structure appears when we consider the surface with the cut disk instead of the puncture. This additional parameter may be identified with the radius of the disk and the corresponding vertex is naturally one- differential that should be integrated over this parameter to get the "hole cutting operator" that we are looking for:

$$V_H = \int dRU_H(R)\,. \tag{3.10}$$

This description uses the explicit parameterization of the moduli space of conformal structures. One could propose the invariant definition of this operator which does not use the explicit parameterization. Consider the contour L on the 2d surface and let V_L be the operator which force the fields in the functional integral to have zero normal derivative on the contour L. The the coordinate independent analog of (3.10) would be given by the integral over the contours:

$$V_H^{\mathrm{inv}} = \int dL V_h(L)\,. \tag{3.11}$$

Perturbations around this vacuum are quite different from the closed string states and may be naturally described in terms of open string states. Note also that this new vacuum is not invariant with respect the symmetries of the closed string vacuum. In particular the gauge symmetry of the B field does not leave it invariant. This leads to the conclusion that in this new vacuum we have the spontaneous breaking of the symmetry and thus some kind of Higgs type effect.

The coordinates in the vicinity of this new vacuum, as usual, have a flavor of the radial coordinates. There are degrees of freedom connected with the symmetries of the theory. The most obvious example is the open string abelian gauge field which is in a sense one of the parameters of the closed string gauge (BRST) transformations. These fields are similar to the angular variables. In particular they are defined up to some identification. In the case of the standard angular variable it is the identification:

$$\theta \sim \theta + 2\pi \tag{3.12}$$

while in the case of the gauge fields it is a gauge transformation:

$$A \sim A + d\phi\,. \tag{3.13}$$

Obviously this gauge symmetry is an artifact of the parameterization and shows up only around the non-trivial vacuum.

The role of the radial coordinates plays the open string tachyon which is invariant with respect to the closed string gauge transformations. In the open string vacuum the expectation value of every "radial" variable is non-zero and it is tempting to conclude that the operator (3.11) is just the open string constant tachyon operator.

The presented picture rises the following question. In the previous section we have argued that the open string states become infinitely massive in the closed string vacuum. But here we move in the opposite direction and looking for the open strings in terms of perturbative closed string theory. We believe that the answer is in the subtleties of the short-distance description of the world-sheet. The open string modes which are infinitely massive in the closed string vacuum are responsible for the non-smooth deformations of the world-sheets and thus are formally absent (have infinite mass) in the perturbative first quantized closed string states. The insertion of the hole cutting operator just change the space of states drastically and the new degrees of freedom are brought about. Note in connection with this the whole question of open/closed string transformation is known to be deeply related with short-distance behaviour of the world-sheet QFT (see e.g. [22]).

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Let us remark that the description of the open string vacuum in terms of the summation over the surfaces is not quite appropriate. We are looking at the theory in the "shifted" vacuum (open strings) using basically the description in terms of world-sheets natural for another vacuum (closed strings).

What is the right framework for the expansion around open string vacuum is not quite clear. The only hint we have is the AdS/CFT correspondence where we are looking at the regime when V_H operator dominates in the action [23].

One could consider these two vacua from another point of view. Consider the open string theory in the background of the constant open string tachyon mode $T = T_0$. The 2d theory is not conformal due to the term on the boundary. This boundary term would give the factor for each boundary:

$$Z \sim e^{-RT_0}\,, \tag{3.14}$$

where R is the length of the boundary.

According to the sigma-model approach one should fix the conformal factor of the metric and integrate over the moduli of conformal structure. At the solutions of the equation of motions the answer does not depend on the choice of the conformal factor due to conformal invariance and away from conformal point it can be compensated by the field-redefinition. Thus we could fix the conformal factor as we want. Let us take some boundary to be of unit length, then we have the overall damping factor of the string amplitudes with the holes. Thus we may conclude that at the new vacuum $T_0 = \infty$ and there should not be any boundary at all.

In terms of above mentioned "hole cutting" operator on could say that at the new string vacuum $T = \infty$ the coefficient in front of this operator in the 2D action is zero. Thus there are no holes and no open strings in this vacuum.

4. String field theory approach

The connection of the open string fields with the parameters of the gauge transformations of the closed strings was important part of the arguments presented above. In this section we test this connection in the framework of the open/closed string field theory constructed by Zwiebach [21]. We will follow closely the notations of [21].

The basic structure used in this construction is Batalin-Vilkovisky (BV) algebra. This structure make possible the correct quantization of the theory. Let Ψ and Φ be closed string and open string wave functions. To define the structure of BV algebra one should define the odd brackets coming from the odd symplectic form and Δ operator acting on this functionals. It was demonstrated in [21] that one could use the product structure on the space of open and closed string functionals:

$$\{A, B\}_{o/c} = \{A, B\}_{\text{open}} + \{A, B\}_{\text{closed}}\,. \tag{4.1}$$

The full action:

$$S(\Phi, \Psi) = S(\Phi)_o + S(\Psi)_c + S(\Phi, \Psi)_{\rm int} \tag{4.2}$$

should satisfy the full quantum BV-equation:

$$\hbar \Delta S + \frac{1}{2}\{S, S\} = 0\,. \tag{4.3}$$

Now consider the classical properties of the BV geometry. The classical part of this equation is:

$$\{S, S\} = 0\,. \tag{4.4}$$

Given the odd symplectic structure on the space one has the algebra of symplectic transformations leaving this symplectic structure invariant. Locally these transformations are generated by the hamiltonian functions U:

$$\delta A = \{A, U\}\,. \tag{4.5}$$

In particular these transformations leave invariant the condition (4.4) and thus transforms the action functional in the field theory into another one suitable for quantization. In general it is not the gauge transformation of the theory because the action functional is not necessary invariant. Gauge transformations are given by the hamiltonian functions of the special form:

$$U = \{f, S\}\,, \tag{4.6}$$

where f is an arbitrary function. It is easy to verify that the invariance of the action functional is the direct consequence of the equation (4.4) .

The general structure of the action functional for the open/closed string theory may be described as follows. The action functional for the open string has the form:

$$S_o(\Phi) = \frac{1}{2}\langle \Phi, Q\Phi\rangle + \langle \Phi^3\rangle + \langle \Phi^4\rangle + \cdots\,, \tag{4.7}$$

where the first term is the free open string action. It is the solution of the open string BV equation (closed strings dropped out):

$$\{S_o, S_o\}_o = 0\,. \tag{4.8}$$

It defines the structure of A_∞ algebra on the open string functionals.

Similarly, the closed string action has the form:

$$S_o(\Psi) = \frac{1}{2}\langle \Psi, Q\Psi\rangle + \langle \Psi^3\rangle + \langle \Psi^4\rangle + \cdots \tag{4.9}$$

and is the solution of the closed string BV equation:

$$\{S_c, S_c\}_c = 0\,. \tag{4.10}$$

(The L_∞ structure on the closed strings.) The important point for our discussion is the appearance of the following second term in the interaction part of the action:

$$S(\Psi, \Phi)_{\text{int}} = \langle \Psi \rangle + \langle \Psi | \Phi \rangle + \langle \Psi | \Phi^2 \rangle + \cdots . \tag{4.11}$$

It was shown in [21] that this coupling between the close and open strings is essential to get the integral over the full moduli space of conformal structures of the surfaces with boundaries in the perturbative expansion of the string field theory.

Consider now the a gauge transformation with the following hamiltonian function:

$$U = \{\langle \eta | \Psi \rangle, S\}, \tag{4.12}$$

where the η is the closed string ghost one state. Taking into account the expressions for the parts of the action (4.7), (4.9), and (4.11) we have

$$U = -\langle Q\eta | \Psi \rangle + \langle \eta | \Phi \rangle + \cdots . \tag{4.13}$$

Here dots are instead of various non-linear terms. Now it is obvious that the gauge symmetry of the full open/closed string action has the form:

$$\delta\Phi = \{\Phi, U\} = (\eta)_o + \cdots \tag{4.14}$$

$$\delta\Psi = \{\Psi, U\} = -Q\eta + \cdots . \tag{4.15}$$

We introduce the notation $(\eta)_o$ here to stress that it is the non-trivial projection of the closed string sector to the open string sector.

From the transformations (4.14) and (4.15) we may conclude that there is the gauge symmetry in the theory of open and closed strings which is the gauge transformation in the closed string sector and the shift in the open string sector. The whole machinery of BV formalism guarantees that this first order transformations could be correctly completed up to the full non-linear symmetry of the theory.

At the end we would like to note that in the framework of [21] the arguments about the suppressing the boundaries by the factor e^{-RT_0} mentioned previously becomes well defined. Just because all string vertexes used in [21] have stubs at the limit $T_0 \to \infty$ closed string part hopefully makes a leading contribution.

5. On the possible generalizations

One could look at the tachyon in the closed string theory from the same perspective. There should be a new "conformal point" at $T \to \infty$. At the new vacuum the 2d surfaces are summed with the coefficient:

$$Z \sim e^{-T_0 A}, \tag{5.1}$$

where A is the area of the surface. Probably the same arguments lead to the conclusion that 2d surfaces should shrink to the point and we leave with closed 3d surface (if any).

One could test these ideas in the case of analog of M-theory and "little" string theories. In the latter case we have 2-branes with the boundaries on the other branes. And the condensation of the tachyon may lead to annihilation of these branes.[5]

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A. On the choice of coordinates

Here we would like to discuss some of the question of regularisation dependence (choice of coordinates in the space of fields) for background independent open string field theory lagrangian on the example of tachyon. The importance of the right choice of the coordinates to have the metric on the fields of the canonical form was already stressed in the main part of the text. Expansion of partition function in the derivatives of tachyon field and gauge field was performed long ago in the original paper on sigma model approach in [25]. Here we will more closely follow the line of the reasoning in [3]. We start with the description of the coordinates, used in [3] for the derivation of the tachyon action up to two derivative terms and then consider the generalization to the case of the abelian gauge fields. In this approximation the general expressions for β-function an the partition function may be written as follows:

$$\beta^T(X) = a_0(T) + a_1(T)\partial T + a_2(T)\partial^2 T + a_3(T)(\partial T)^2 + \cdots \tag{A.1}$$

$$Z = \int d^{26}X e^{-T(X)}(1 + b(T)(\partial T)^2 + \cdots)\,. \tag{A.2}$$

To make considerations more simple we begin with natural coordinates in the sigma model approach and find corresponding β and Z. In these coordinates the boundary action has the form $\int d\sigma T(X)$.

It is rather obvious that zero mode of the tachyon enters in the partition function as the overall prefactor e^{-T_0}. Therefore we may infer that $b(T) = \text{const.}$, $a_0(T) = T$,

[5]In a sense this is an analog of [6] for closed strings-boundary of (2+1)d membrane; the possibility that the closed string field theory could be constructed in analogous to [6] fashion via the 3d surface with boundary was suggested long ago [24].

a_1 = const., a_2 = const., a_3 = const. and T_0 enters the β-function as an additive term:

$$\beta_T(T) = T_0 + (T_0 - \text{independent terms})\,. \tag{A.3}$$

The partition function and β-function for the quadratic profile $T = a + u/4X^2$ were calculated in [6] . Using these explicit calculations we conclude that: $a_0(T) = T, b = a_1 = a_3 = 0, a_2 = 2$.

Substituting the β-function:

$$\beta_T(T) = T + 2\Delta T \tag{A.4}$$

and partition function:

$$Z = \int d^{26}X e^{-T(X)} \tag{A.5}$$

in the basic equation:

$$S = -\beta^i \partial_i Z + Z \tag{A.6}$$

we have the following action:

$$S = \int dX^{26} e^{-T}[2(\partial T)^2 + (T+1)] \tag{A.7}$$

with the corresponding equations of motion:

$$e^{-T}(T + 4\Delta T - 2(\partial T)^2) = 0\,. \tag{A.8}$$

The equations of motion are related to the β-functions by the metric G_{ij} on the space of fields.

$$\partial_i S = G_{ij}\beta^j \tag{A.9}$$

Comparing (A.8) with the (A.4) we find:

$$G(\delta_1 T, \delta_2 T) = \int dX e^{-T}(\delta_1 T \delta_2 T - 2(d\delta_1 T)(d\delta_2 T))\,. \tag{A.10}$$

This metric is rather complex and not obviously is an expansion of some invertible metric on the space of fields. Now we make a change of coordinates leading to more simple form of the metric (note that these new coordinates were used in [3] in order to write down the tachyon action exactly following the above line of reasoning).

We make this field redefinition in two steps.

First we consider the linear T-term. Note that T-linear term in β function is invariant with respect to the scaling of the tachyon field ($\beta^i\partial_i$ is a vector field) while the linear part of the equation of motions (or quadratic part of the action) is not.

Consider the new coordinates on the space of the tachyon configurations:

$$T \to T - \partial^2 T\,. \tag{A.11}$$

We have the following expressions:

$$\beta_T(X) = T + 2\Delta T \tag{A.12}$$

$$Z(T) = \int e^{-T}(1 + (\partial T)^2 + \cdots) \tag{A.13}$$

$$S = \int dX e^{-T}(T+1)((\partial T)^2 + 1)\,. \tag{A.14}$$

Obviously, in terms of [6] this is equivalent to different regularisation of the Green function at coincident points.

Now the linear terms in the equation of motion are proportional to the linear terms in the β-function with the simple coefficient e^{-T}. Note that it is obviously invertible for finite T.

Finally consider the new coordinate T_*:

$$T_* = T - (\partial T)^2\,. \tag{A.15}$$

In terms of the new coordinate we have ($b = 0$):

$$Z(T_*) = \int dX e^{-T_*}(1 + \cdots)\,. \tag{A.16}$$

The corresponding β-function may be obtained from the condition of the invariance:

$$\beta(T(X))\frac{\delta}{\delta T(X)} = \beta_*(T_*(X))\frac{\delta}{\delta T_*(X)} \tag{A.17}$$

and has the following form:

$$\beta_*(T_*) = T_* + 2\Delta T_* - (dT_*)^2\,. \tag{A.18}$$

From this we obtain the final action (in the new coordinates T_*):

$$S = \int dX e^{-T_*}[(\partial T_*)^2 + (T_* + 1)] \tag{A.19}$$

with the equations of motion:

$$\frac{\partial}{\partial T_*} S(T_*) = e^{-T}(T_* + 2\Delta T_* - (dT_*)^2) = e^{-T_*}\beta(T_*) = 0\,. \tag{A.20}$$

This is the form of the action given in [3].

Thus we have demonstrated that the action (A.19) is connected with the action (A.7) by the field redefinition:

$$T \to T - \partial^2 T + (\partial T)^2 + \cdots \tag{A.21}$$

and that in the variables (A.19) the metric which relates equations of motion and world-sheet β-function has the simple form of multiplication by e^{-T_*} as opposed to the one for (A.7).

Now let us include into the consideration the abelian gauge field. Using the same approximation we consider the part of the lagrangian which depends polynomially on the gauge field stress-tensor (up to second order) and does not depend on its derivatives. This approximation is similar to taking into account the first two terms of the expansion of Born-Infeld lagrangian.

At this approximation we have the following expressions for the partition function and β-functions generalizing (A.1) and (A.5)

$$\beta^T(X) = T + 2\Delta T + c_1(T)F^2 \cdots \tag{A.22}$$

$$\beta^A(X)_\nu = c_2\partial^\mu T F_{\mu\nu} + \tilde{c}_2\partial^\mu F_{\mu\nu} + \cdots \tag{A.23}$$

$$Z = \int d^{26}X e^{-T(X)}(1 + c_3(T)F^2 + \cdots)\,. \tag{A.24}$$

Analogously to the case of the pure tachyon we could deduce that c_i's are independent on T.

Using the definition of the action functional (A.6) one has in the necessary approximation:

$$S(T,A) = \int dX^{26}e^{-T}((T+1) + 2(\partial T)^2 + (c_1+c_3)F^2 + c_3TF^2 + \cdots) \tag{A.25}$$

with the corresponding equations of motion for the tachyon being:

$$e^T\frac{\delta S}{\delta T} = -T - 2\Delta T - c_1F^2 - c_3TF^2 + \cdots\,. \tag{A.26}$$

Consider new tachyon field:

$$T^* = T + c_3F^2 \tag{A.27}$$

$$A^* = A\,. \tag{A.28}$$

Taking into account the property of the covariance for the beta function:

$$\beta^T\frac{\partial}{\partial T} + \beta^A\frac{\partial}{\partial A} = \beta^{T^*}\frac{\partial}{\partial T^*} + \beta^{A^*}\frac{\partial}{\partial A^*} \tag{A.29}$$

we find that the new beta function is:

$$\beta^{T^*}(X) = -T^* - 2\Delta T^* + (c_1+c_3)F^2\,. \tag{A.30}$$

Thus, in the new coordinates the action and the equations of motion are:

$$S(T^*,A^*) = \int dX^{26}e^{-T^*}((T^*+1) + 2(\partial T)^2 + (c_1+c_3)F^2 + \cdots) \tag{A.31}$$

$$e^T\frac{\delta S}{\delta T^*} = -T^* - 2\Delta T^* - (c_1+c_3)F^2 + \cdots\,. \tag{A.32}$$

Combining (A.21), (A.27) and (A.28) we have the lagrangian:

$$S(T,A) = \int dX^{26} e^{-T}((T+1) + (\partial T)^2 + (c_1 + c_3)F^2 + \cdots)\,. \tag{A.33}$$

The coefficients $c_1 = 0$ and $c_3 = 1/4$ obtained in the appropriate regularisation scheme could be read off from the formula (3.22) in [26] (see also [9] and [10]). Covariantizing the action with respect to diffeomorphisms and $B_{\mu\nu}$ field gauge transformations we end up with the following result:

$$S(T,A) = \int dX^{26} \sqrt{G} e^{-T}((T+1) + ||dT||^2 + \frac{1}{4}||B - F||^2 + \cdots)\,. \tag{A.34}$$

One shall note that the choice of coordinates as in lagrangian (A.25) for $c = 0, c_3 = 1/4$ would lead to gauge field dependence through $\int V(T)F^2$ which is natural to expect since it is second order term in expansion of Born-Infeld action replacing the metric in $\int V(T)\sqrt{G}$. Our principle of choice of coordinates leads to (A.34) instead.

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